DARPA HAWK

DTIC

AD-A167 773

RADAR CROSS-SECTION OF DAMPED CYLINDERS AND ANISOTROPIC CYLINDERS

Technical Progress Report

ITEM 0002

March 17, 1986

Prepared by

Richard Holland

for

Office of Naval Research

Under

Contract Number N00014-85-C-0852

A04946

Applied Physics, Inc.

5353 Wyoming Blvd., NE, Suite 3 Albuquerque, New Mexico 87109
(505) 823-2334

DARPA 4941

RADAR CROSS-SECTION OF DAMPED CYLINDERS AND ANISOTROPIC CYLINDERS

Technical Progress Report

ITEM 0002

March 17, 1986

Prepared by

Richard Holland

for

Office of Naval Research

Under

Contract Number N00014-85-C-0852

A04946

Applied Physics, Inc.

5353 Wyoming Blvd., NE, Suite 3 Albuquerque, New Mexico 87109
(505) 823-2334

Technical Progress Report

Contract Number N00014-85-C-0852

Radar Cross-Section of Damped Cylinders
and Anisotropic Cylinders

This quarter, we have generalized the TDFD RCS code to handle materials characterized by:

(1) Lossy, frequency-dependent dielectrics,
(2) Magnetic loss and conductivity,
(3) Anisotropy in the xy plane, and
(4) Embedded resistive or reactive sheets.

Our treatment of frequency dependence goes beyond what had been the state of the art in TDFD codes; the frequency dependence here may be anisotropic and may extend across a molecular resonance. The techniques for doing this are documented in the attached article, "Time-domain treatment of Maxwell's equations in frequency dependent media." This article was written for the March conference in Monterey and represents work entirely inspired by the present contract.

A second computer code has also been written to compute the RCS of a circular cylinder composed of an arbitrary number of concentric shells, each characterized by an arbitrary σ, σ^*, ϵ and μ. This code uses the expansion of plane wave in cylindrical harmonics and matching of harmonic coefficients at each interface. It is described in the attached note, "Scattering from a layered dielectric cylinder."

Code-code comparisons between the TDFD and cylindrical harmonic codes have been run for a perfect conductor .5 m in radius, bare and covered by a damper .5 m thick. The damper has properties $\epsilon/\epsilon_0 = \mu/\mu_0 = 1$; $\sigma = \epsilon\sigma^*/\mu = 4 \times 10^{-3}$. These values were selected to give a skin depth on the order of the damper thickness at frequencies (50 - 500 MHz) for which calculations were run. The TDFD code utilized square cells 4 cm on each side or 25 cells to a cylinder diameter.

While our TDFD code only treats the TM case, we are able to simulate the TE problem through appeal to duality. In particular, the previously described code-code comparison was rerun with the cylinder perfectly magnetically conducting ($\sigma^* = \infty$, $\sigma = 0$); and the damper unchanged.

The following four figures present code-code results for the four possible combinations of electric and magnetic conducting cylinders with and without damping shells.

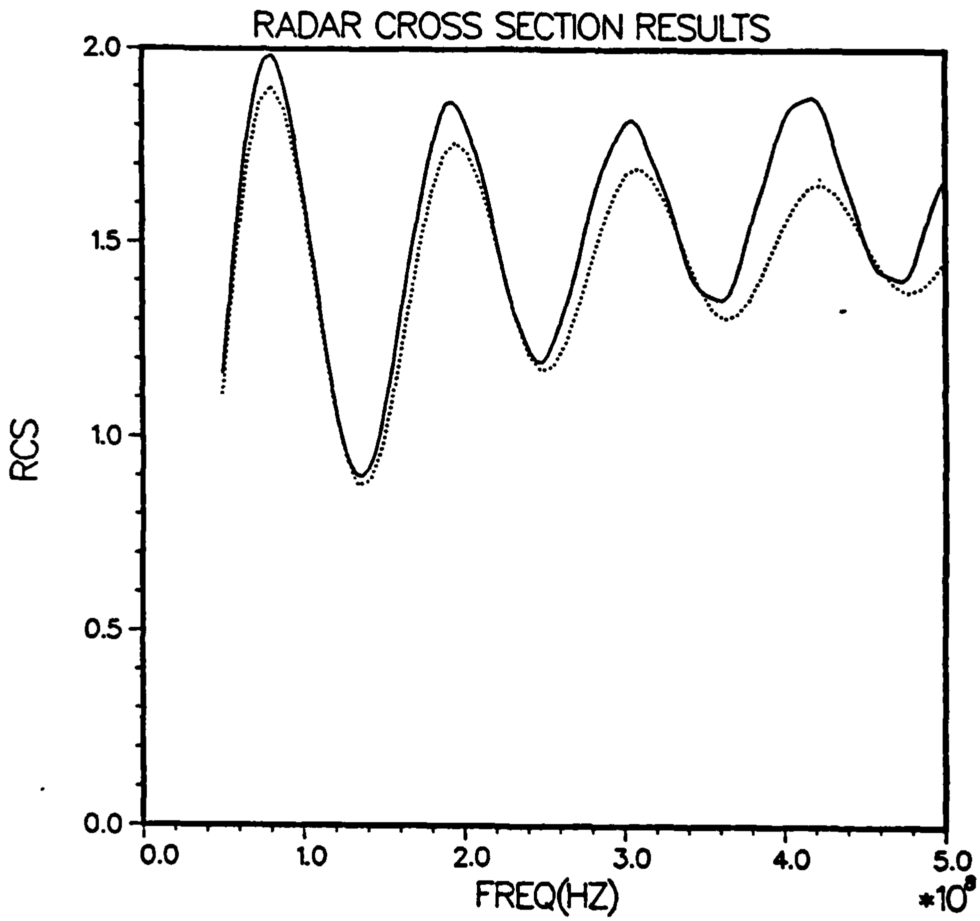

Figure 1. RCS of a bare cylinder, .5 m in radius, electrically perfectly conducting for TM illumination ($\underline{H}$ along axis). Solid curve is TDFD result; dotted curve is cylindrical harmonic result.

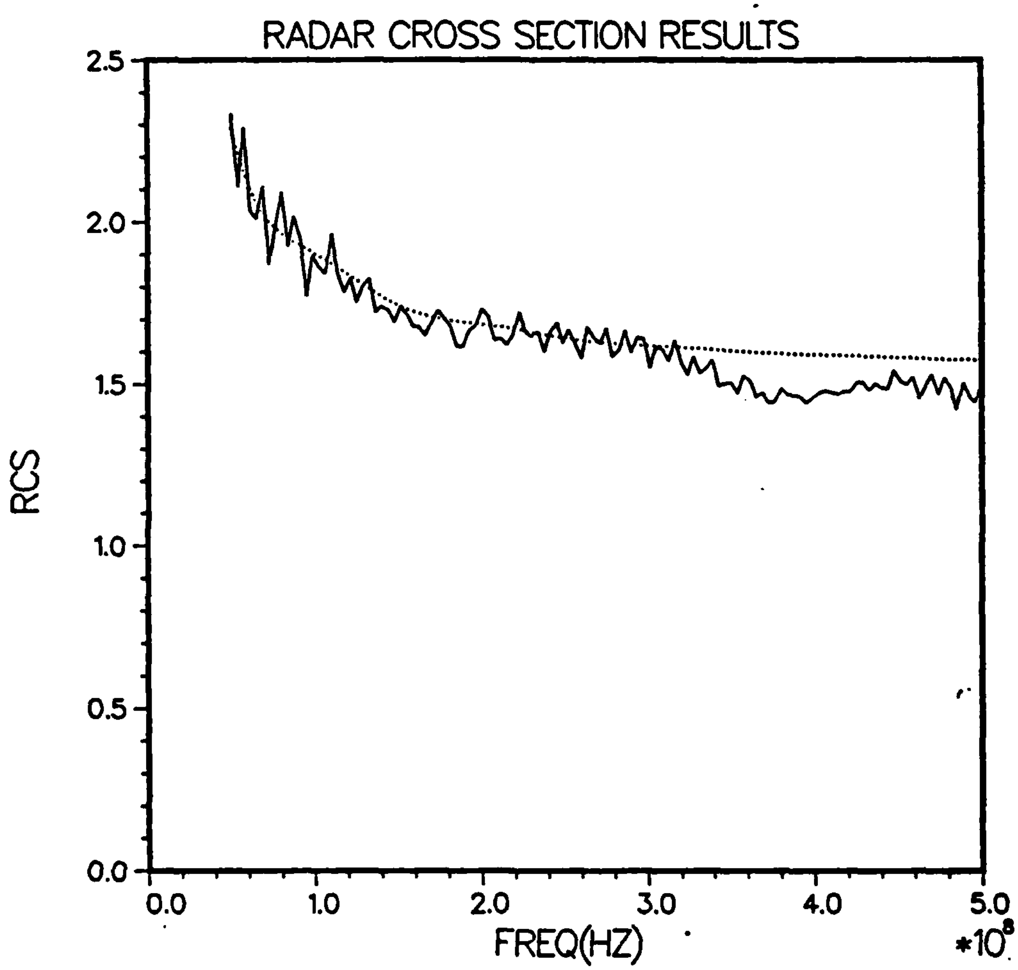

Figure 2. RCS of a bare cylinder, .5 m in radius, magnetically perfectly conducting for TM illumination (H along axis). Solid curve is TDFD result; dotted curve is cylindrical harmonic result.

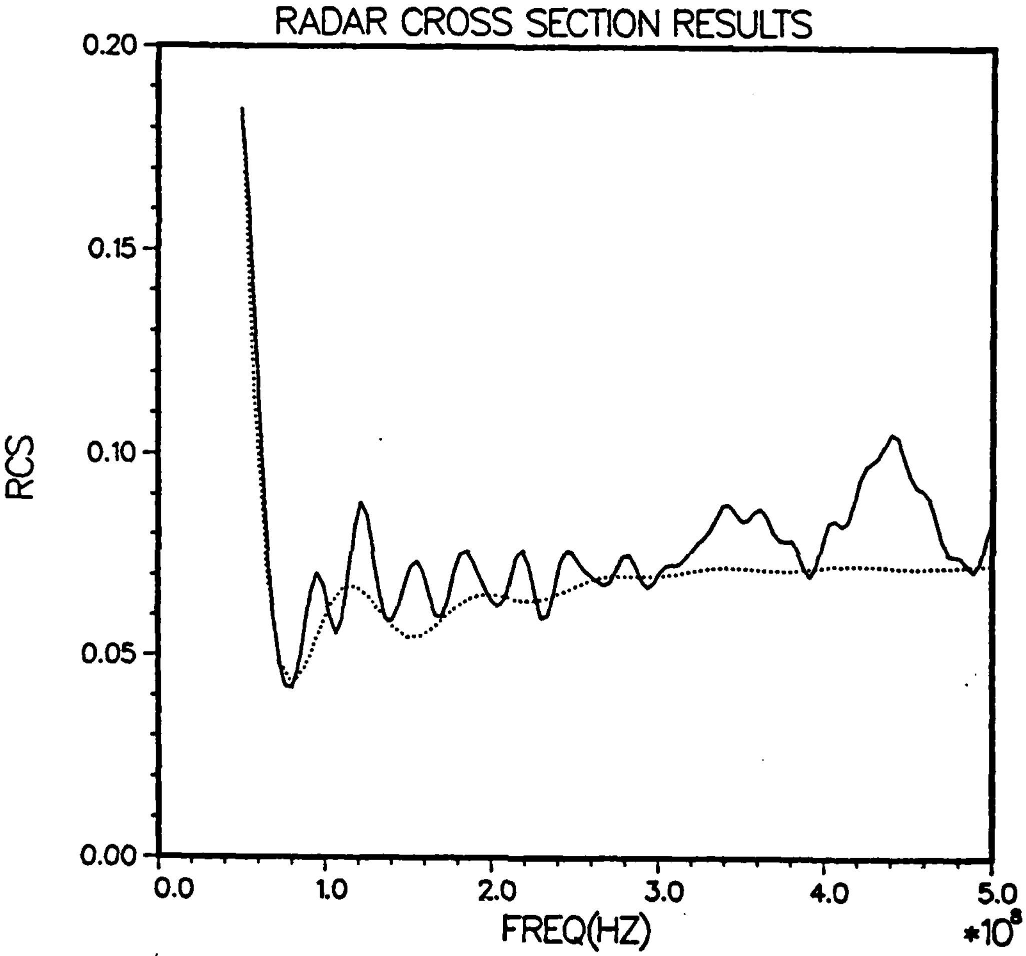

Figure 3. RCS of a damped cylinder, .5 m in radius, electrically perfectly conducting for TM illumination (H along axis). Solid curve is TDFD result; dotted curve is cylindrical harmonic result.

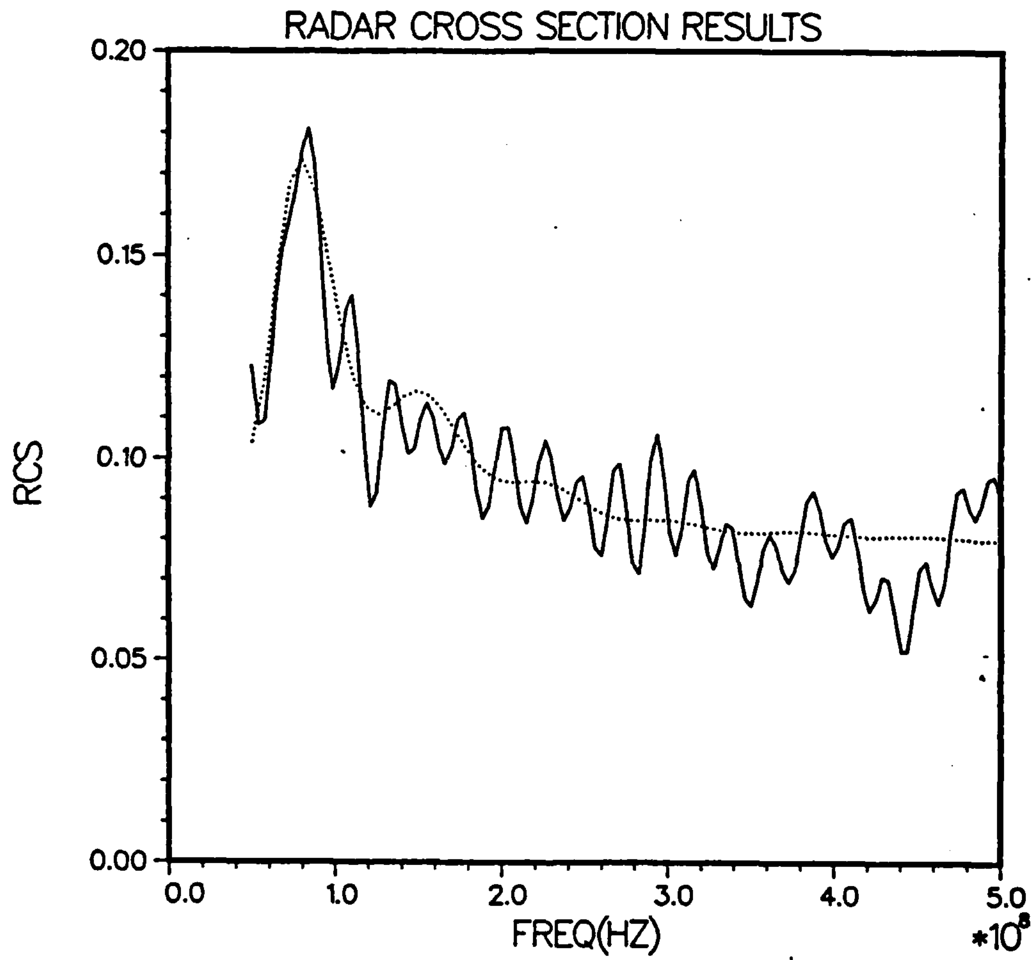

Figure 4. RCS of a damped cylinder, .5 m in radius, magnetically perfectly conducting for TM illumination (H along axis). Solid curve is TDFD result; dotted curve is cylindrical harmonic result.

Scattering from a Layered Dielectric Cylinder

This note discusses scattering of a plane wave by a circular dielectric cylinder composed of concentric layers of different materials. Let us assume there are N layers, with layer i characterized by ϵ_i, μ_i, σ_i, σ_i^* and outer radius a_i. We shall here treat the TM case ($\underline{H}$ along the cylinder axis; $\underline{E}$ transverse), although the TM problem is nearly identical mathematically.

Assume the incident wave is propagating in the + y direction,

$$\underline{H}^{inc}(\underline{r},t) = Y_0 \underline{i}_z e^{i(k_0 r \sin\phi - \omega t)} \tag{1}$$

$$\underline{E}^{inc}(\underline{r},t) = - \underline{i}_x e^{i(k_0 r \sin\phi - \omega t)} \tag{2}$$

Here, $Y_0 = \sqrt{\frac{\epsilon_0}{\mu_0}}$ is the admittance of free space. The same symbol will subsequently be used to designate Neuman functions, but context should keep the meaning unambiguous. Additionally, k_0 is the free space wavenumber, $\omega\sqrt{\epsilon_0\mu_0} = \omega/c$.

Equation (1) may be expanded in cylindrical harmonics,

$$\underline{H}^{inc}(\underline{r},t) = Y_0 \underline{i}_z \sum_{n=-\infty}^{\infty} J_n(k_0 r) e^{in\phi}$$

$$= Y_0 \underline{i}_z \left[J_0(k_0 r) + \sum_{n=2,2}^{\infty} 2J_n(k_0 r)\cos n\phi + \sum_{n=1,2}^{\infty} 2iJ_n(k_0 r)\sin n\phi \right] \tag{3}$$

In the future, it will be useful to designate the coefficients of these harmonics as a_n^{inc};

$$a_o^{inc} = 1$$

$$a_n^{inc} = 2 \qquad n > 0,\ \text{even}$$

$$a_n^{inc} = 2i \qquad n\ \text{odd} \tag{4}$$

Since $\nabla \times \underline{H}^{inc} = - j\omega\epsilon_0\underline{E}^{inc}$, the cylindrical harmonic expansion for $\underline{E}^{inc}$ becomes

$$\underline{E}^{inc}(\underline{r},t) = \frac{iY_0}{\omega\epsilon_0}\left[- \frac{\underline{i}_r}{r}\left(\sum_{n=0,2}^{\infty} a_n^{inc}J_n(k_0r)n \sin n\phi - \sum_{n=1,2}^{\infty} a_n^{inc}J_n(k_0r)n \cos n\phi\right)\right.$$

$$\left. - \underline{i}_\phi k_0\left(\sum_{n=0,2}^{\infty} a_n^{inc}J_n'(k_0r)\cos n\phi + \sum_{n=1,2}^{\infty} a_n^{inc}J_n'(k_0r)\sin n\phi\right)\right] \tag{5}$$

The innermost material will include the cylinder axis. Thus, only Bessel functions of the first kind are permitted in the solution there:

$$\underline{H}^1(\underline{r},t) = Y_1\underline{i}_z\left[\sum_{n=0,2}^{\infty} a_n^1J_n(k_1r)\cos n\phi + \sum_{n=1,2}^{\infty} a_n^1J_n(k_1r)\sin n\phi\right] \tag{6}$$

$$\underline{E}^1(\underline{r},t) = \frac{iY_1}{\omega\epsilon_1}\left[- \frac{\underline{i}_r}{r}\left(\sum_{n=0,2}^{\infty} a_n^1J_n(k_1r)n \sin n\phi - \sum_{n=1,2}^{\infty} a_n^1J_n(k_1r)n \cos n\phi\right)\right.$$

$$- \mathbf{i}_\phi k_1 \left[\sum_{n=0,2}^{\infty} a_n^1 J_n'(k_1 r) \cos n\phi + \sum_{n=1,2}^{\infty} a_n^1 J_n'(k_1 r) \sin n\phi \right] \tag{7}$$

where Y_1 is the admittance of medium 1

$$Y_1 = \sqrt{\frac{\epsilon_1 + j\sigma_1/\omega}{\mu_1 + j\sigma_1^*/\omega}} \tag{8}$$

and k_1 is the wavenumber of medium 1,

$$k_1 = \omega\sqrt{(\epsilon_1 + j\sigma_1/\omega)(\mu_1 + j\sigma_1^*/\omega}) \tag{9}$$

The N-1 concentric shells will permit solutions of both kinds. Thus, in region i, $1 < i \le N$, $a_n^1 J_n(k_1 r)$ of eqs. (6)-(9) becomes replaced by

$$a_n^1 J_n(k_1 r) \rightarrow a_n^i J_n(k_i r) + b_n^i Y_n(k_i r) \tag{10}$$

Finally, in free space outside the cylinder, the first Hankel function is the only permitted solution for the scattered field. Thus, in this region, $a_n^{inc} J_n(k_0 r)$ is replaced by

$$a_n^{inc} J_n(k_0 r) \rightarrow a_n^{scat} H_n^{(1)}(k_0 r) \tag{11}$$

in eqs. (3)-(5).

The boundary conditions at each interface are that ϵE_r, E_ϕ and H_z be continuous. It turns out that the first and third of these conditions are equivalent. Thus, matching of coefficients at the innermost interface leads to

$$a_n^1 Y_1 J_n(k_1 a_1) - a_n^2 Y_2 J_n(k_2 a_1) - b_n^2 Y_2 Y_n(k_2 a_1) = 0$$

$$a_n^1 J_n'(k_1 a_1) - a_n^2 J_n'(k_2 a_1) - b_n^2 Y_n'(k_2 a_1) = 0 \tag{12}$$

Matching of coefficients at any other interface except the outer boundary of the cylinder gives

$$a_n^{i-1} Y_{i-1} J_n(k_{i-1} a_{i-1}) + b_n^{i-1} Y_{i-1} Y_n(k_{i-1} a_{i-1}) - a_n^i Y_i J_n(k_i a_{i-1}) - b_n^i Y_i Y_n(k_i a_{i-1}) = 0$$

$$a_n^{i-1} J_n'(k_{i-1} a_{i-1}) + b_n^{i-1} Y_n'(k_{i-1} a_{i-1}) - a_n^i J_n'(k_i a_{i-1}) - b_n^i Y_n'(k_i a_{i-1}) = 0 \tag{13}$$

Finally, the boundary condition at the outermost surface is

$$a_n^N Y_N J_n(k_N a_N) + b_n^N Y_N Y_n(k_N a_N) - a_n^{scat} Y_0 H_n^{(1)}(k_0 a_N) = a_n^{inc} Y_0 J_n(k_0 a_N)$$

$$a_n^N J_n'(k_N a_N) + b_n^N Y_n'(k_N a_N) - a_n^{scat} H_n^{(1)\prime}(k_0 a_N) = a_n^{inc} J_n'(k_0 a_N) \tag{14}$$

For each azimuthal harmonic, eqs. (12)-(14) comprise a set of 2N linear equations in 2N unknowns. The associated matrix is five-banded, and extremely easy to solve by Gaussian elimination. (The main diagonal and first diagonal off each side of the main is full. The second diagonal off each side of the main is half zeros.)

The quantities of interest in RCS evaluation are the a_n^{scat}'s. A two-dimensional bislatic RCS is defined by

$$RCS(\phi) = 2\pi \lim_{r\to\infty} \left| \frac{E^{scat}(\phi)\sqrt{r}}{E^{inc}} \right|^2 \qquad (15)$$

The scattered electric field (neglecting the reactive radial component) is

$$\underline{E}^{scat}(\underline{r},t)$$

$$= - \underline{i}_\phi \left(\sum_{n=0,2}^{\infty} a_n^{scat} H_n^{(1)\prime}(k_0 r)\cos n\phi + \sum_{n=1,2}^{\infty} a_n^{scat} H_n^{\prime}(k_0 r)\sin n\phi \right) \qquad (16)$$

Here, use is made of the identity $k_0 Y_0 = \omega\epsilon_0$.

For large arguments, Hankel functions have the asymptotic limit

$$H_n^{(1)}(k_0 r) \to \sqrt{\frac{2}{\pi k_0 r}}\, e^{i(kr - \frac{2n+1}{4}\pi)} \qquad (17)$$

Substitution of eqs. (16) and (17) in (15) then yields the RCS in terms of the a_n^{scat}:

$$\mathrm{RCS}(\phi) = \frac{4}{k_0}\left| \sum_{n=0,2}^{\infty} a_n^{scat}(-i)^n \cos n\phi + \sum_{n=1,2}^{\infty} a_n^{scat}(-1)^n \sin n\phi \right|^2 \tag{18}$$

TIME-DOMAIN TREATMENT OF MAXWELL'S EQUATIONS IN FREQUENCY DEPENDENT MEDIA

Richard Holland
Applied Physics, Inc.
5353 Wyoming Blvd., NE, Suite 3
Albuquerque, NM 87109

ABSTRACT

Longmire and Longley (DNA report 3167F, 1973) have described a method for representing certain types of frequency-dependent media in time-domain finite difference codes. They assumed the media could be described by a series of relaxation phenomena, one phenomenon per decade. Each phenomenon contributes a "current" to the $\nabla \times H$ equation, although these "currents" do not conveniently fit either the concept of conduction or displacement.

Recent discussions with ONR personnel have raised interest in more general types of media characterization. Specifically, there is now a desire to input the SEM or Prony parameters of a medium's σ and ϵ to time-domain codes. In other words, we now wish to work with the actual poles and residues of σ and ϵ rather than assuming the poles are uniformly spaced one per decade. The present work describes how this may be done.

It also goes beyond the work of Longmire and Longley in permitting use of complex as well as real poles. Prony "currents" associated with complex poles obey a temporally second-order differential equation, as opposed to the Prony "currents" of real poles. In both this formulation, and of the Longmire-Longley formulation, the latter obey temporally first-order differential equations.

In actual time-domain finite-difference codes, it is most exact to evaluate the Prony "currents" as well as the electric fields at the same spatial and temporal points. In the past, this simultaneous solution has not been implemented, due to the peculiar coupling of the equations for electric fields and Prony "currents". For the case of all poles real, we here also describe how to perform this exact solution using state theory;

Work was sponsored by Office of Naval Research and DARPA under Contract No. N00014-85-C-0852

i.e., by exponentiating the matrices coupling all the electric fields and Prony "currents".

INTRODUCTION

Consider a medium with anisotropic, frequency-dependent electrical properties. The electrical response of such a material may be fairly generally described by

$$\nabla \times \underline{H} = \underline{J}(t) + \underline{J}_f(t) \tag{1}$$

where $\underline{J}_f(t)$ is a forced current and

$$\underline{J}(t) = \underline{\underline{\sigma}}_0 \cdot \underline{E}(t) + \underline{\underline{\epsilon}}_\infty \cdot D\underline{E}(t) + \int_{\infty}^{t} \underline{\underline{K}}(t-t') \cdot D\underline{E}(t')dt' \tag{2}$$

with D indicating the time-derivative operator. In this formulation, $\underline{\underline{\sigma}}_0$, $\underline{\underline{\epsilon}}_\infty$, and $\underline{\underline{K}}(t-t')$ are second-rank tensors.

The frequency-domain form of eq. (2) is

$$\underline{J}(\omega) = \left[\underline{\underline{\sigma}}_0 + i\omega\underline{\underline{\epsilon}}_\infty + i\omega \int_0^\infty e^{-i\omega u}\underline{\underline{K}}(u)du\right] \cdot \underline{E}(\omega) \tag{3}$$

Separation of eq. (3) into real and imaginary parts gives representations for the frequency-dependent conductivity and permittivity tensors,

$$\underline{\underline{\sigma}}(\omega) = \underline{\underline{\sigma}}_0 + \mathrm{Re}\left[i\omega\int_0^\infty e^{-i\omega u}\underline{\underline{K}}(u)du\right] \tag{4}$$

$$\underline{\epsilon}(\omega) = \underline{\epsilon}_\infty + \mathrm{Re}\left[\int_0^\infty e^{-i\omega u}\underline{K}(u)\,du\right] \tag{5}$$

Longmire and Longley[1] have considered the scalar version of this formulation for the special case when K(u) can be expressed as an exponential series,

$$K(u) = \sum_{m=1}^{M} a_m e^{-\beta_m u} \tag{6}$$

For this situation, eq. (2) may be rewritten

$$\underline{J}(t) = \sigma_0\underline{E}(t) + \epsilon_\infty D\underline{E}(t) + \sum_{m=1}^{M} a_m\underline{J}_m(t) \tag{7}$$

with

$$\underline{J}_m(t) = e^{-\beta_m t}\int_\infty^t D\underline{E}(t')e^{\beta_m t'}\,dt' \tag{8}$$

Equation (8) is equivalent to the differential equation

$$D\underline{J}_m(t) = D\underline{E}(t) - \beta_m\underline{J}_m(t) \tag{9}$$

Longmire and Longley assumed that materials could be represented by the exponential series of eq. (6) with one term for each decade of frequency over the spectrum of interest. This is equivalent to doing a Prony expansion of K(u) [or $\sigma(\omega)$ and $\epsilon(\omega)$] with the poles forced to be spaced at

$$s_m = 10^{m+m_0} = \beta_m \tag{10}$$

While this assumption has been claimed to be reasonably accurate for wet soil, it would seem generally more correct to determine the poles from a Prony analysis of the medium's measured frequency-dependent characteristics. This is especially likely to be true if the material exhibits rapid variation in σ and ϵ with frequency.

STATE THEORY APPLICATIONS

Let us first assume the Prony analysis reveals no complex-conjugate pole pairs. In general, the $\underline{\underline{a}}_m$ will be second rank tensors, but the β_m will only be scalars. Then for every pole, each component of $\underline{J}_m$ will obey

$$DE_i - DJ_{mi} - \beta_m J_{mi} = 0 \qquad m = 1 - M,\ i = 1 - 3 \tag{11}$$

Additionally, the tensor form of eq. (7) gives

$$\epsilon_{\infty ij} DE_j + \sigma_{0ij} E_j + \sum_{m=1}^{M} a_{mij}\, J_{mj} = J_j \qquad i,j = 1 - 3 \tag{12}$$

where

$$J_j = (\nabla \times \underline{H} - \underline{J}_f)_j \tag{13}$$

Equations (11) and (12) constitute a set of 3(M+1) coupled first order differential equations.

If anisotropy and frequency dependence were not present, the usual method of numerical solution would be explicit time-domain finite differencing. In this method, E and H evaluation points alternate both spatially and temporally using a well-tested leapfrog arrangement.[2-4] In this arrangement, no two equations are coupled, and $E_x^{n+1/2}(I,J,K)$ means E_x evaluated at $((I + 1/2)\Delta X, J\Delta Y, K\Delta Z, (n + 1/2)\Delta t)$.

However, the present system of equations requires the three E_j's and 3M J_{mj}'s all to be evaluated simultaneously. While this cannot be done using conventional time-domain finite differencing, state theory does indicate an appropriate generalization of time-domain finite differencing.

First, let us consider the case where anisotropy, but not frequency dependence, is present,

$$[\epsilon_\infty]\, D[E] + [\sigma_0][E] = [J] \tag{14}$$

This matrix differential equation has a homogeneous solution

$$[E]_h = e^{-[\epsilon_\infty]^{-1}[\sigma_0]t}[A] \tag{15}$$

and a particular solution

$$[E]_p = [\sigma_0]^{-1}[J] \tag{16}$$

giving a general solution

$$[E] = e^{-[\epsilon_\infty]^{-1}[\sigma_0]t}[A] + [\sigma_0]^{-1}[J] \tag{17}$$

The constant vector [A] may be evaluated at $(n - 1/2)\Delta t$:

$$[E]^{n-1/2} = [A] + [\sigma_0]^{-1}[J]^n \tag{18}$$

This gives the new E-field vector in terms of the old,

$$[E]^{n+1/2} = e^{-[\epsilon_\infty]^{-1}[\sigma_0]\Delta t}[E]^{n-1/2} + \left(1 - e^{-[\epsilon_\infty]^{-1}[\sigma_0]\Delta t}\right)[\sigma_0]^{-1}[J]^n \tag{19}$$

Similar exponential matrix techniques have been reported for time-domain solution of generalized multi-conductor transmission lines.[5] In the previous work, one may see how to evaluate eq. (19) if $[\sigma_0]$ is singular or if $[\epsilon_\infty]^{-1}[\sigma_0]\Delta t$ has arbitrarily large elements. Basically, matrices are exponentiated using the power-series representation of an exponential.

If frequency dependence is present, the [E] vector of eqs. (14)-(19) becomes replaced by

$$[E] \rightarrow \begin{bmatrix} \underline{E} \\ \underline{J}_1 \\ \vdots \\ \underline{J}_M \end{bmatrix} = [E'] \tag{20}$$

The $[\epsilon_\infty]$ matrix becomes

$$[\epsilon_\infty] \rightarrow \begin{bmatrix} \underline{\epsilon_\infty} & \underline{0} & \cdots & \underline{0} \\ \underline{I} & -\underline{I} & \cdots & -\underline{I} \\ \vdots & \vdots & & \vdots \\ \underline{I} & -\underline{I} & \cdots & -\underline{I} \end{bmatrix} = [\epsilon'] \tag{21}$$

and the $[\sigma_0]$ matrix becomes

$$[\sigma_0] \rightarrow \begin{bmatrix} \underline{\sigma}_0 & \underline{a}_1 & \cdots & \underline{a}_M \\ \underline{0} & -\beta_1 \underline{I} & \cdots & \underline{0} \\ \vdots & \vdots & & \vdots \\ \underline{0} & \underline{0} & \cdots & -\beta_M \underline{I} \end{bmatrix} = [\sigma'] \tag{22}$$

Lastly, the forcing vector becomes

$$[J] \rightarrow \begin{bmatrix} \underline{J} \\ \underline{0} \\ \vdots \\ \underline{0} \end{bmatrix} = [\underline{J}'] \tag{23}$$

Then the matrix equation for simultaneous advancement of $\underline{E}$ and the $\underline{J}_m$ is

$$[E']^{n+1/2} = e^{-[\epsilon']^{-1}[\sigma']\Delta t}[E']^{n-1/2} + \left(1 - e^{-[\epsilon']^{-1}[\sigma']\Delta t}\right)[\sigma']^{-1}[J']^{n} \tag{24}$$

In the past, time-domain finite differencing has not often considered anisotropy. Frequency-dependent effects have been included by using the old $\underline{J}_m^{n-1/2}$ to find the new $\underline{E}^{n+1/2}$. (This decouples $\underline{E}$ from the $\underline{J}_m$ in eq. (12).) Then the new $\underline{E}^{n+1/2}$ have been used to find the new $\underline{J}_m^{n+1/2}$ from eq. (11).

TREATMENT OF COMPLEX POLES

If Prony analysis of the material's frequency dependence reveals complex pole pairs, a more general treatment becomes necessary. In this case, $\underline{K}(u)$ will contain terms of the form

$$\underline{K}(u) = \underline{b}_m \sin(\gamma_m u + \phi_m) e^{-\beta_m u}$$

$$= \underline{b}_m \cos\phi_m \sin\gamma_m u e^{-\beta_m u} + \underline{b}_m \sin\phi_m \cos\gamma_m u e^{-\beta_m u} \quad (25)$$

The $\underline{J}_m(t)$ of eq. (8) now becomes

$$\underline{J}_m(t) = \int_{\infty}^{t} D\underline{E}(t')(\cos\phi_m \sin\gamma_m(t-t')e^{-\beta_m(t-t')}$$

$$+ \sin\phi_m \cos\gamma_m(t-t')e^{-\beta_m(t-t')})dt'$$

$$= \underline{J}_{mc}(t)\cos\phi_m + \underline{J}_{ms}(t)\sin\phi_m \quad (26)$$

where $\underline{J}_{mc}$ and $\underline{J}_{ms}$ are the parts of $\underline{J}_m$ associated with $\cos\phi_m$ and $\sin\phi_m$, respectively:

$$\underline{J}_{mc}(t) = \int_{-\infty}^{t} D\underline{E}(t')\sin\gamma_m(t-t')e^{-\beta_m(t-t')} dt \quad (27)$$

$$\underline{J}_{ms}(t') = \int_{-\infty}^{t} D\underline{E}(t')\cos\gamma_m(t-t')e^{-\beta_m(t-t')} dt' \quad (28)$$

Differentiation of $\underline{J}_{mc}(t)$ and $\underline{J}_{ms}(t)$ yields

$$D\underline{J}_{mc}(t) = -\beta_m \underline{J}_{mc}(t) + \gamma_m \underline{J}_{ms}(t) \quad (29)$$

$$D\underline{J}_{ms}(t) = D\underline{E}(t) - \beta_m \underline{J}_{ms}(t) + \gamma_m \underline{J}_{mc}(t) \tag{30}$$

These equations can be solved for $\underline{J}_{mc}(t)$ and $\underline{J}_{ms}(t)$ using Heavyside algebra:

$$[(D + \beta_m)^2 + \gamma_m^2]\ \underline{J}_{mc}(t) = \gamma_m D\underline{E}(t) \tag{31}$$

$$[(D + \beta_m)^2 + \gamma_m^2]\ \underline{J}_{ms}(t) = (D + \beta_m)D\underline{E}(t) \tag{32}$$

Thus, $\underline{J}_m(t)$ obeys the differential equation

$$[(D + \beta_m)^2 + \gamma_m^2]\ \underline{J}_m(t) = [\sin\phi_m(D + \beta_m) + \cos\phi_m\gamma_m]D\underline{E}(t) \tag{33}$$

In principle, equations like (33) could be added to the set of equations given by (11) and (12), and the entire ensemble solved by state theory. This approach, however, requires treatment of second-order matrix differential equations of the form

$$[A]D^2[E] + [B]D[E] + [C][E] = [F] \tag{34}$$

The homogeneous solution of this equations includes square roots and complex exponents of matrices; it is much more complicated than eqs. (14) - (19). (To the best of our knowledge, exponential differencing has never been applied even to scalar second-order differential equations.)

Consequently, when Prony analysis of the material data yields complex pole pairs, our present strategy is to fall back to the old technique for dealing with real poles: First find $\underline{E}^{n+1/2}$ using the old $\underline{J}_m^{n-1/2}$. Then use

the new $\underline{E}^{n+1/2}$ and the finite-difference form of eq. (33) to find the new $\underline{J}_m^{n+1/2}$.

REFERENCES

1. Longmire, C.L. and H.J. Longley, "Time Domain Treatment of Media with Frequency-Dependent Electrical Parameters," DNA 3167F, 23 September 1973.

2. Yee, K.S., "Numerical Solution of Initial Boundary Value Problems Involving Maxwell's Equations in Isotropic Media," IEEE Trans. Antennas & Propagat., Vol. AP-14, pp. 302-307, May 1966

3. Longmire, C.L., "State of the Art in IEMP and SGEMP Calculations," IEEE Trans. Nuc. Sci., Vol. NS-22, pp. 2340-2344, December 1975.

4. Holland, R., "THREDE: A Free-Field EMP Coupling and Scattering Code," IEEE Trans. Nuc. Sci., Vol. NS-24, pp. 2416-2421, December 1977.

5. Holland, R., "The EMP Tunnel Code DHTRANS," IEEE Trans. Nuc. Sci., Vol. NS-25, pp. 1404-1411, December 1978.